Volmex Implied Volatility Indices

Kadir Gökhan Babaoğlu

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This paper develops a new methodology to calculate the implied volatility of crypto assets using option contracts from multiple sources. It describes how to consolidate and filter the option data, to calculate the implied variance and to smooth this variance in order to finalize the volatility calculation.

At Volmex Labs, we implemented this methodology to publish the Volmex implied volatility indices which measure a 30-day implied volatility of crypto assets.[1]

1 Introduction

Implied volatility (IV), believed to be the **fear gauge**[2] of investors, simply refers to the volatility that is extracted from (and implied by) options by matching the price of these contracts. It is a single number that combines all the information across available strikes for a given expiry, and measures overall expensiveness of option contracts.

The pricing of options depends not only on the price level of the underlying asset but also on its volatility, which is actually a latent variable. This latent variable (i.e., IV) is not observed directly but can be inferred from the prices of option contracts, and is considered to be the expected volatility from now to the expiration. Thus, IV reflects the option market participants' expectations about the price range of the underlying asset over the life of the contract and it is "forward-looking."

IV calculations can be model-dependent or model-free. Model-dependent IVs have restrictive assumptions on the underlying asset price and other state variable processes (stochastic volatility or jump intensity), whereas model-free IVs work with minimal assumptions on the generating processes of state variables.

The Volmex IV indices use a model-free approach[3] to calculate IV of crypto assets, and reflect the forward-looking market view expressed as an index.

[1] Currently available underlying crypto assets are ETH and BTC

[2] See Whaley (2000).

[3] See Demeterfi et al. (1999)

2 Methodology

The Volmex IV calculation methodology does not depend on any model or any data source. It consolidates option data from various option exchanges and creates a global option order book. It uses variance-swap replication method to calculate implied variance in a model-free way and then uses the exponentially-weighted moving average (EWMA) method to smooth the raw variance. Smoothing provides the advantage of de-noising the index without missing a trend. It is parameterized in a balanced fashion so that it captures the emerging trends but at the same time ignores noisy observations.

In order to infer the implied volatility from a set of option contracts, the Volmex IV methodology follows the four steps below. Each of them is crucial for the calculation of the final implied volatility value:

1. **Data consolidation**: The first step of this methodology is to gather all the option and future contracts' data from multiple sources and create a global order book of options and risk-free curve.
2. **Filtering**: The second step filters the out-of-the-money (OTM) call and put option contracts to be used in the variance calculation.
3. **Calculation**: Raw implied variance calculation using variance swap replication is performed by computing the contributions of each of the selected options, summing them up and making the adjustments to get the raw value, which is only based on the current selection of option contracts.
4. **Smoothing**: The last step calculates the final IV as the square root of exponentially-weighted moving average (EWMA) of raw implied variances. The resulting smoothed IV reduces the noise and provides gradual updating even when the raw values in the previous step become unstable.

3 Data consolidation

The Volmex IV methodology employs multiple data sources and each source provides raw data in their own format. These data need to be extracted and standardized in the same format before getting consolidated.

3.1 Data extraction

Source data should at least have bid/ask quotes and mark prices[4] for each option contract, together with a timestamp that indicates when the data are collected.

In addition, some providers have corresponding forward prices associated with each option contract. These prices are mark prices and we use them when they

[4] i.e., the reference price that providers use in P&L calculations

are available. Section 3.3.1 describes the consolidation process of future prices to create a yield curve that we use in the Volmex IV calculation.

3.2 Underlying asset price level

We assume that each exchange provides the current value of the underlying asset price and its timestamp. Given the price and timestamp, we use the most recent price of all available prices as the global price and use it in Volmex IV calculations.

3.3 Future contracts: Interest rate term structure

Option exchanges offer futures trading as well, and these data could be retrieved when we receive option data.

If data is attached to the option data, i.e., mark prices of corresponding futures or forward prices, then we do not retrieve futures quotes to keep the time discrepancy at minimum between option and future, and build the interest rate term structure (risk-free curve) as in Section 3.3.1.

However, if data is not available in the option data set, we retrieve futures quotes and process as described in Section 3.3.2.

3.3.1 Using mark prices

We calculate the interest rate using the global price level mentioned in Section 3.2 using the formula below.

$$F = S \times \mathrm{e}^{r \times t} \tag{1}$$

where F is the forward price, S is the underlying price level, t is year-to-maturity (YTM) and r is the interest rate. Thus, the implied interest rate is

$$r_{imp} = \frac{\ln F - \ln S}{t}. \tag{2}$$

Once interest rates are calculated for all the future contracts, we calculate the average interest rate for each expiry. This gives us the yield curve for the underlying asset.

3.3.2 Using bid-ask quotes

If future contracts data comes separately, we use bid and ask quotes to consolidate the option data and then build the yield curve based on the formula in equation 2.

To consolidate the data, we collect bid/ask quotes and pick the maximum bid and minimum ask for each maturity. We compute mid-price (which is the average of bid and ask prices) as the forward price. After that, using equation 2, we calculate the interest rates for each maturity therefore finding the yield curve needed for further steps of Volmex IV calculation.

3.4 Option contracts

Processing the options data that we retrieve from various exchanges has two steps. First, we extract the information from each of the exchange, and then combine them as a single data set of options. Details of these two steps are discussed in the following sections.

3.4.1 Source data

1. Select near and next-term options:
    - *Near-term*: Options with longest maturity that is less than or equal to index maturity
    - *Next-term*: Options with shortest maturity that is more than index maturity
2. Eliminate invalid quotes under the following scenarios:
    - Negative bid/ask spread[5]
    - Mark price is out of bid/ask range[6]
    - Mark price is not positive.

We perform these operations for the data of each exchange, and next section uses them to create a consolidated option data set to use in the remaining steps.

3.4.2 Consolidate data

1. Combine all data in one set. Since the option data are normalized in the previous step, we do not have any issues to concatenate these data sets.
2. Select near and next-term options: Each provider may have different option expiry dates, so we need to perform this step in the aggregate level as well
    - *Near-term*: Options with longest maturity that is less than or equal to index maturity
    - *Next-term*: Options with shortest maturity that is more than index maturity

[5] Ask price is less than bid price
[6] Mark price is less than bid price or greater than ask price

3. Consolidate bid and ask quotes for each option contract with same type[7], strike and expiry:
   - Select quotes: maximum of bids and minimum of asks available.
   - Select mark prices: mark price of the option with smallest bid/ask spread.
4. Eliminate invalid quotes under the following scenarios:
   - Negative bid/ask spread.[8]
   - Mark price is not positive.
5. Eliminate the quotes with large spreads:
   - Calculate bid spread as the difference between mark price and bid price. Set the bid spread to zero if negative.
   - Calculate ask spread as the difference between ask price and mark price. Set the ask spread to zero if negative.
   - Calculate spread as the sum of bid and ask spreads.
   - Calculate the maximum allowed spread (MAS) as the minimum of bid and ask spreads, multiplied by `SPREAD_MULTIPLIER`[9].
   - Calculate the global maximum spread (GMS) as `SPREAD_MIN`[10] multiplied by `SPREAD_MULTIPLIER`
   - Remove the quote if its spread is greater than both GMS and MAS.
6. Calculate mid-price as the average of bid and ask prices.
7. Calculate interest rate[11] using yield curve introduced in Section 3.3.

Once all data are consolidated in a single data set, we move to the next stage where we filter out-of-the-money (OTM) option contracts.

4 Filtering

1. Index maturity is set at 30 days and year convention is 365 days, which gives index YTM as $30/365 \approx 0.082192$ years.
2. Calculate the implied forward price of the strike that has minimum absolute mid-price difference between call and put options, for near and next-term options: $F_{imp} = K + F \times (C - P)$ where F is the forward price, C is the call option price, P is put option price, and both options are quoted in the amounts of underlying.

[7] i.e., call or put
[8] Ask price is less than bid price
[9] Currently set at 4.0
[10] Equal to tick size; currently set at 0.0005 for BTC and ETH
[11] Use linear interpolation and extrapolate further tenors at the last observed values

3. Set the largest strike that is less than the implied forward F_{imp} as ATM strike K_{ATM} for near and next-term options.

4. Select OTM options with respect to K_{ATM} for each set of near and next-term options and set their price as mid-price. If both call and put options are selected for the same strike (i.e., K_{ATM}), then take the average of them.

5. Select the options with strikes greater than K_{min} and less than K_{max},

$$K_{min} = F_{imp}/\texttt{RANGE\_MULT}$$
$$K_{max} = F_{imp} \times \texttt{RANGE\_MULT}$$

   where F_{imp} ensures that the range of strikes change dynamically.[12]

6. Sort the options on strikes and eliminate the options after observing two consecutive bid prices that are equal or less than minimum bid threshold.[13]

5 Raw implied variance calculation

Once filtering is done, near and next-term option contracts are plugged in the implied variance calculation that replicates variance swaps.

The implied variance formula[14] for near and next-term options is,

$$\sigma_{i,t}^2 = \frac{1}{T_i}\left(2\sum_j w_{i,j}V(K_j) - \left(\frac{F_\mathrm{i}}{K_{i,\mathrm{ATM}}} - 1\right)^2\right)$$

where $w_{i,j} = \mathrm{e}^{r_i T_i}\dfrac{\Delta K_j}{K_j}$, $i \in \{\mathrm{NEAR}, \mathrm{NEXT}\}$, F_i is the implied forward price, $K_{i,ATM}$ is the ATM strike level, $V(K)$ is the price[15] of OTM option with strike K, and T_i is years to expiry.

Once $\sigma^2_{\mathrm{NEAR},t}$ and $\sigma^2_{\mathrm{NEXT},t}$ are calculated following the formula above, implied variance at the index maturity is interpolated using the weighting scheme below:

$$\omega_{\mathrm{NEAR},t} = \frac{(T_\mathrm{NEXT} - T_\mathrm{INDEX})/T_\mathrm{INDEX}}{(T_\mathrm{NEXT} - T_\mathrm{NEAR})/T_\mathrm{NEAR}} \tag{3}$$

$$\omega_{\mathrm{NEXT},t} = \frac{(T_\mathrm{INDEX} - T_\mathrm{NEAR})/T_\mathrm{INDEX}}{(T_\mathrm{NEXT} - T_\mathrm{NEAR})/T_\mathrm{NEXT}} \tag{4}$$

[12] Currently `RANGE_MULT` is set to 3.0
[13] Equal to tick size
[14] See Demeterfi et al. (1999)
[15] i.e., mid-price

where T_{INDEX} has been set as 30/365.

Thus, the raw value of implied variance at the index maturity is,

$$\sigma^2_{\text{RAW},t} = \omega_{\text{NEAR},t}\sigma^2_{\text{NEAR},t} + \omega_{\text{NEXT},t}\sigma^2_{\text{NEXT},t}$$

which is calculated continuously at a frequency that we acquire new data.

6 Smoothing

Raw implied variance could suffer from noise since it is calculated when a new data set is available[16] and it might be over-sensitive to sudden changes.

6.1 Exponentially-weighted moving average (EWMA)

To eliminate the noise in raw implied variance and capture the fundamental trends in the market, an EWMA of raw implied variance is employed as shown below:

$$\sigma^2_{\text{SMOOTH},t} = \lambda\sigma^2_{\text{SMOOTH},t-1} + (1-\lambda)\sigma^2_{\text{RAW},t} \quad (5)$$

where $\sigma^2_{\text{SMOOTH},t-1}$ is the previous value of the smoothed implied variance and its weight is the smoothing parameter λ.

6.2 Half-life: Choosing the smoothing parameter

The degree of smoothing is adjusted by the smoothing parameter λ. Equation 5 could be defined recursively, as below:

$$\sigma^2_{\text{SMOOTH},t} = \lambda^\tau\sigma^2_{\text{SMOOTH},t-\tau} + (1-\lambda)\sum_{i=0}^{\tau-1}\lambda^\tau\sigma^2_{\text{RAW},t-i}$$

and we can find the smoothing parameter value that implies a specific half-life[17] τ using the derivation below,

$$\lambda^\tau = \frac{1}{2} \Rightarrow \lambda = \mathrm{e}^{-\ln(2)/\tau}.$$

Above equation gives $\lambda = 0.97716$ when we set the half-life to 30 seconds and implied variance calculation is performed every second,

$$0.97716^{30\text{sec}\times 1\text{calculation/sec}} = 0.97716^{30} \approx 0.5$$

which simply means that the weight of current smooth variance will be halved every 30 seconds (i.e., 30 calculations given that calculations are performed every second) if we choose λ as 0.97716.

[16]Currently it is every second
[17]Currently it is set to 30 seconds

6.3 Volmex implied volatility

The Volmex IV of a crypto asset x, abbreviated as `xVIV`,[18] is the squared root of the smoothed implied variance that has been calculated, multiplied by 100, since it is expressed as percentage points:

$$\texttt{xVIV}_t = 100 \times \sqrt{\sigma^2_{\text{SMOOTH},t}}$$

7 Conclusion

This paper introduces a new methodology to calculate the implied volatility of crypto assets at a given expiry/tenor, and Volmex uses this methodology at 30 days maturity to calculate each IV index. It has the novel features of: i) consolidating option and futures data across multiple exchanges to create a global option book, and ii) smoothing using exponentially-weighted moving averaging, which takes every observation into account while keeping the final index moving smoothly.

References

K. Demeterfi, E. Derman, M. Kamal, and J. Zou. A guide to volatility and variance swaps. *The Journal of Derivatives*, 6(4):9–32, 1999.

R. E. Whaley. The investor fear gauge. *The Journal of Portfolio Management*, 26(3):12–17, 2000.

[18] e.g. EVIV is for the Volmex IV of ETH, and BVIV is the Volmex IV of BTC

Appendix A Exceptions

The Volmex IV calculation depends on the health of the data and some exchanges might experience difficulties around 8 a.m. UTC for several reasons. In addition, option market makers may not always quickly react to the sudden changes in the market and pull their quotes for a small amount of time for revaluation and re-quoting purposes.

These scenarios may cause significant reduction in the amount of available option quotes in the exchanges and may affect the calculation of raw implied variance.

A.1 Scenarios

Two scenarios are considered as exceptions:

1. Failure in calculating the raw implied variance numerically
2. Raw implied variance less than Black-Scholes at-the-money (ATM) implied variance (BSIV)

In the above scenarios, we use BSIV and Volatility Tail Index (VTI) to calculate the raw implied variance.

A.2 Variables

Exceptions could happen to three variables:

1. Near-term raw implied variance
2. Next-term raw implied variance
3. Raw implied variance (30-day)

Each of these variables have their own BSIV and VTIs.

A.3 Black-Scholes ATM Implied Volatility (BSIV)

When we calculate the reference ATM strike, K_{ATM} for near and next-term option contracts, we select the five[19] contracts closest to K_{ATM} and calculate Black-Scholes implied volatility (BSIV) of these contracts, select two smallest of the available BSIVs, and then calculate the average of them. This gives us near and next-term BSIVs and we use the weighting scheme in Equations 3 and 4 to calculate the 30-day BSIV. All BSIVs are also expressed in percentage points.

[19] If all of the selected quotes are invalid, try 10 first and then 15.

A.4 Volatility Tail Index (VTI)

VTI measures the percentage difference between the Volmex IV and BSIV. Since the Volmex IV uses OTM options and BSIV is only informative about ATM options, the ratio of these two would indicate how tails are priced differently than central strike region (i.e., ATM). We calculate raw VTIs for near-term, next-term and 30-day expiries, and then smooth them using the same value of λ that we use in Equation 5.

$$\begin{aligned} \texttt{VTI}_{raw,t} &= 100 \times (\texttt{xVIV}_t / \texttt{BSIV}_t - 1) \\ \texttt{VTI}_{smooth,t} &= \lambda \times \texttt{VTI}_{smooth,t-1} + (1-\lambda) \times \texttt{VTI}_{raw,t} \end{aligned}$$

where `xVIV` refers to the Volmex IV of crypto asset x.

A.5 Calculation

In case of any exception in the calculation of raw implied variance (of near-term, next-term, or 30-day), we use BSIV and VTI values to calculate raw variance as shown below:

$$\sigma^2_{\mathrm{RAW},t} = \left[\frac{\texttt{BSIV}_t}{100} \times \left(1 + \frac{\texttt{VTI}_{smooth,t-1}}{100}\right)\right]^2$$