VCORR: Volmex Spot-Volatility Correlation Indices

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This paper develops a new methodology to calculate the correlation between implied volatility of crypto assets and the underlying spot price.[1] *At Volmex Labs, we implemented this methodology to publish the Volmex Correlation Indices.*[2]

1 Introduction

Spot-volatility correlation, the correlation between implied volatility and the underlying asset return, plays an important role in understanding the current market state in addition to implied volatility for several reasons.

First of all, rising implied volatility could be attributed not only to market crash or panic but also happens when the market is highly greedy and retail demand on calls pressures option prices upwards. Therefore, it would be crucial to capture this dimension of market state as an indicator.

Secondly, spot-volatility correlations could be used as a predictor of future implied volatility and underlying returns since it captures the market regimes more accurately. Thus, it could help both sophisticated and retail traders have a better idea on the option markets' current state.

2 Methodology

VCORR calculation has two main steps where the first step is to calculate the raw covariance matrix and the second step is to smooth these values using exponentially-weighted moving average.

Before discussing these two steps in detail, first we define change in price and change in volatility variables. We use log difference since it can transform the

[1] i.e., Spot-volatility correlation
[2] Currently available underlying crypto assets are ETH and BTC

probability distribution of original variables into more normal-looking distributions:

$$r_{S,t} = \ln S_t - \ln S_{t-\Delta t} \tag{1}$$

$$r_{IV,t} = \ln IV_t - \ln IV_{t-\Delta t} \tag{2}$$

where S_t is the price level of a crypto asset at time t, IV_t is the implied volatility at time t and Δt is the time step which is the time difference between two observations.

Next, we define vectors of change in price and change in implied volatility:

$$r_{S,t,N} = \left[r_{S,t} \dots r_{S,t-(N-1)\times\Delta t}\right]^\top \tag{3}$$

$$r_{IV,t,N} = \left[r_{IV,t} \dots r_{IV,t-(N-1)\times\Delta t}\right]^\top \tag{4}$$

where $r_{S,t,N}$ is a vector that keeps the most recent N values of the changes in price and $r_{IV,t,N}$ is that of implied volatility.

2.1 Raw Covariance Matrix Calculation

Covariance matrix of $r_{S,t,N}$ and $r_{IV,t,N}$, say $\Sigma_{t,N}$, consists of covariance between $r_{S,t,N}$ and $r_{IV,t,N}$, variance of $r_{S,t,N}$ and variance of $r_{IV,t,N}$:

$$\Sigma_{t,N} = \begin{bmatrix} \operatorname{cov}\left(r_{S,t,N}, r_{S,t,N}\right) & \operatorname{cov}\left(r_{S,t,N}, r_{IV,t,N}\right) \\ \operatorname{cov}\left(r_{S,t,N}, r_{IV,t,N}\right) & \operatorname{cov}\left(r_{IV,t,N}, r_{IV,t,N}\right) \end{bmatrix} \tag{5}$$

These covariance vaules are calculated as shown below:

$$\operatorname{cov}\left(r_{S,t,N}, r_{IV,t,N}\right) = \operatorname{E}\Big[\left(r_{S,t,N} - \operatorname{E}\left[r_{S,t,N}\right]\right) \times \left(r_{IV,t,N} - \operatorname{E}\left[r_{S,t,N}\right]\right)\Big] \tag{6}$$

$$\operatorname{cov}\left(r_{IV,t,N}, r_{IV,t,N}\right) = \operatorname{var}\left(r_{IV,t,N}\right) \tag{7}$$

$$\operatorname{cov}\left(r_{S,t,N}, r_{S,t,N}\right) = \operatorname{var}\left(r_{S,t,N}\right) \tag{8}$$

where var(x) is the variance of the values in the vector x.

2.2 Smoothing the Covariance

Since we use covariance matrix to calculate the correlation, any issues in the covariance matrix could affect the value of the correlation.

Raw covariance could suffer from noise since it is calculated when a new data set is available[3] and it might be over-sensitive to sudden changes.

[3] Currently it is every minute

2.2.1 Exponentially-weighted moving average (EWMA)

To eliminate the noise in raw covariance and capture the fundamental trends in the market, an EWMA of raw covariance is employed as shown below:

$$\Sigma_{\text{SMOOTH},t,N} = \lambda \times \Sigma_{\text{SMOOTH},t-1,N} + (1-\lambda) \times \Sigma_{t,N} \tag{9}$$

where $\Sigma_{\text{SMOOTH},t,N}$ is the previous value of the smoothed covariance matrix and its weight is the smoothing parameter λ.

The smoothed covariance matrix is 2-by-2 and has the following elements:

$$\Sigma_{\text{SMOOTH},t,N} = \begin{bmatrix} \sigma_{r_{S,t,N},r_{IV,t,N}} & \sigma^2_{r_{S,t,N}} \\ \sigma^2_{r_{IV,t,N}} & \sigma_{r_{S,t,N},r_{IV,t,N}} \end{bmatrix} \tag{10}$$

2.2.2 Half-life: Choosing the smoothing parameter

The degree of smoothing is adjusted by the smoothing parameter λ, and we can find the smoothing parameter value that implies a specific half-life[4] τ using the derivation below,

$$\lambda^\tau = \frac{1}{2} \Rightarrow \lambda = \mathrm{e}^{-\ln(2)/\tau}.$$

Above equation gives $\lambda = 0.9885$ when we set the half-life to 1 hour and covariance calculation is performed every minute,

$$0.9885^{60\text{min}\times 1\text{calculation/min}} = 0.9885^{30} \approx 0.5$$

which simply means that the weight of current smooth covariance matrix will be halved every 1 hour (i.e., 60 calculations given that calculations are performed every minute) if we choose λ as 0.9885.

2.2.3 Volmex Spot-volatility Correlation

xVCORR of a crypto asset x, abbreviated as `xCORR`,[5] is the correlation that has been calculated from the covariance matrix, multiplied by 100, since it is expressed as percentage points:

$$\texttt{xVCORR}_t = 100 \times \frac{\sigma_{r_{S,t,N},r_{IV,t,N}}}{\sqrt{\sigma^2_{r_{S,t,N}} \times \sigma^2_{r_{IV,t,N}}}}$$

3 Conclusion

This paper introduces a new methodology to calculate the spot-volatility correlation of crypto assets. It has the novel features of smoothing using exponentially-weighted moving averaging and using smoothed covariance matrix to calculate final correlation values.

[4] Currently it is set to 1 hour

[5] e.g. EVCORR is for the Volmex Spot-volatility Correlation for ETH, and BVCORR is for BTC