

VBULL & VBEAR

Volmex Bull & Bear Implied Volatility Indices

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***Abstract.** This paper introduces a novel methodology for calculating semi-volatilities, defined as the square root of annualized semi-variances, where semi-variances represent the squared return variations below or above a specified threshold. The methodology is applied to cryptocurrency assets, with a focus on calculating option-implied (risk-neutral) semi-volatilities for both positive and negative return variations.*

1. Introduction

Volatility is a key measure of risk in financial markets, commonly used to assess the uncertainty associated with an asset's price movements. Traditionally, volatility is measured using standard deviation, which treats both positive and negative price fluctuations equally. However, this symmetric treatment of returns does not account for the fact that market participants often view upward and downward risks very differently. For instance, while investors may welcome upside volatility, which signals the potential for gains, they are usually more concerned about downside volatility, which represents the risk of losses. This asymmetric risk preference is particularly pronounced in the context of options markets, where hedgers and speculators seek to protect against specific types of price movements. To address this issue, we propose a novel methodology for calculating semivolatilities, allowing for the separate measurement of upward (bullish) and downward (bearish) risks.

The unique characteristics of cryptocurrency markets make them ideal candidates for this refined analysis of volatility. Crypto assets are known for their extreme price fluctuations and high levels of volatility. Furthermore, the growing market for cryptocurrency derivatives, such as options and futures, has created a demand for more sophisticated volatility measures that can provide deeper insights into market behavior. Volatility indices, such as the VIX in traditional markets or Bitcoin Volmex Implied Volatility (BVIV) index in the crypto space, are useful but give an overall picture instead of differentiating between positive and negative return variations. By introducing Volmex Bull and Bear Implied Volatility Indices (VBULL & VBEAR), we aim to fill this gap, offering market participants a more granular understanding of crypto-asset volatility and the associated risks.

This paper presents the theoretical framework and practical application of these indices, built on the concept of semivariance. Semivariance allows us to measure the dispersion of returns either above or below a specific threshold, typically the mean or zero. In doing so, we can isolate the volatility attributed to positive returns (bull volatility) and negative returns (bear volatility). We apply this methodology to calculate option-implied semivolatilities, providing a risk-neutral perspective on both upward and downward market movements.

2. Semivariance and Semivolatility

The core concept behind the VBULL & VBEAR is the idea of semivariance, which provides a more detailed view of market risk by separating upward and downward return variations. In this section, we provide more information on semivariances.

Semivariance is a measure of the dispersion of all returns that fall below or above a specified threshold, typically the mean or zero. Unlike variance, which considers both upward and downward deviations, semi-variance focuses only on downside or upside risk. Mathematically, downside semivariance is expressed as

$$\text{Downside Semivariance} = \frac{1}{N} \sum_{i=1}^N \max(0, m - r_i)^2$$

$$\text{Upside Semivariance} = \frac{1}{N} \sum_{i=1}^N \max(0, r_i - m)^2$$

where r_i represents the returns, and the threshold m .

The threshold is typically set to zero, which gives that the downside semivariance is the average of squared negative returns, and that the upside semivariance is the average of squared positive returns. Thus their total is equal to variance:

$$\text{Variance} = \text{Downside Semivariance} + \text{Upside Semivariance} \text{ (if } m = 0\text{)}$$

Bull and Bear Volatility

To capture the asymmetry in risk preferences, we introduce two distinct measures of semi-volatility:

- **Bull Volatility (VBULL):** VBULL captures the volatility associated with upward movements
- **Bear Volatility (VBEAR):** VBEAR captures the volatility associated with downward movements

These measures are particularly useful in the context of options pricing and risk management, where the concern over losses (bearish movements) might differ from the concern over gains (bullish movements).

3. Methodology

The model-free implied variance (MFIV) formulated in Britten-Jones and Neuberger (2000) is given as

$$MFIV = \frac{2\exp(r \times T)}{T} \times \left(\int_0^F \frac{Put(K,T)}{K^2} dK + \int_F^\infty \frac{Call(K,T)}{K^2} dK \right).$$

The equation is identical to the formulation by Demeterfi et al (1999), which is used in its discrete-time form and also employed by Volmex Implied Volatility Indices.

Redefining the Semivariances

In the previous formulation in Section 2, the downside and upside semivariances are calculated independently, and their sum is equal to the total variance. However, this approach does not inherently place the total variance between the two semivariances, making it difficult to directly compare downside and upside risks. To address this, we propose a redefinition of the semivariances, which improves the interpretability of the measures and allows for a more intuitive comparison between upward (bull) and downward (bear) volatilities.

To achieve this, we modify the semivariance definitions using a set of out-of-the-money (OTM) put and call options. We mirror the left tail (represented by OTM put options) to replicate the right tail, in order to create a more informative set of option prices for downside risk. We do the same using the right tail for upside risk. These mirrored sets of option prices enable us to treat the upside and downside risks symmetrically around the forward price of the underlying asset. This mirroring is done by transforming the strike prices into log-moneyness, defined as:

$$m = \ln K - \ln F$$

where K represents the strike price and F is the forward price of the asset. This transformation ensures that we are operating in a consistent framework where both tails of the distribution are comparable.

If we express the option prices in functions of log-moneyness, then the new set of options that we use in the semivariance calculation becomes:

$$\begin{aligned} \overline{Call}(-m, T) &:= Put(m, T) \text{ if } K \geq F \\ \overline{Put}(-m, T) &:= Call(m, T) \text{ if } K < F. \end{aligned}$$

Using these mirrored option prices, we redefine the downside and upside semi-variances as follows:

$$\begin{aligned} \text{Downside Semivariance} &= \frac{2\exp(r \times T)}{T} \times \left(\int_0^F \frac{Put(K,T)}{K^2} dK + \int_F^\infty \frac{\overline{Call}(K,T)}{K^2} dK \right), \\ \text{Upside Semivariance} &= \frac{2\exp(r \times T)}{T} \times \left(\int_0^F \frac{\overline{Put}(K,T)}{K^2} dK + \int_F^\infty \frac{Call(K,T)}{K^2} dK \right), \end{aligned}$$

where $\overline{Call}(K, T)$ and $\overline{Put}(K, T)$ are new sets of OTM option prices that we define by replicating the OTM half of the implied future returns distribution in the in-the-money (ITM) half.

Calculation of the Semivariances

Once we have new sets of option prices that are derived from OTM put (left-tail) or call (right-tail) options only, we follow the implied volatility calculation by Volmex Implied Volatility Indices, as discussed the following subsections:

1. Calculate near and next term semivariances

Using the variance swap replication, we can calculate the near and next-term semivariances,

$$\sigma_{i,s}^2 = \frac{1}{T_i} \left(2 \sum_j w_{i,j} V_s(K_j, T_i) - \left[\frac{F_i}{K_{i,ATM}} - 1 \right]^2 \right)$$

where $w_{i,j} = \exp(r_i \times T_i) \frac{\Delta K_j}{K_j^2}$, $i \in \{near, next\}$, F_i is the implied forward price, $K_{i,ATM}$ is the ATM strike price, $V_s(K_j, T_i)$ is the price of OTM option with strike K_j , and T_i is years to expiry, $s \in \{bull, bear\}$ is the set of options where bear set is derived from only OTM puts and bull set is from only OTM calls.

Following the formula above, we calculate $\sigma_{near, bull}^2$, $\sigma_{near, bear}^2$, $\sigma_{next, bull}^2$ and $\sigma_{next, bear}^2$.

2. Calculate raw semivariance of 30-day maturity

Semivariance at 30-day maturity is interpolated using the weighting scheme below:

$$w_{near} = \frac{(T_{next} - T_{ref}) / T_{ref}}{(T_{next} - T_{near}) / T_{near}}$$

$$w_{next} = \frac{(T_{ref} - T_{near}) / T_{ref}}{(T_{next} - T_{near}) / T_{next}}$$

where T_{next} is the maturity of next term options, T_{near} is the maturity of near term options, and T_{ref} has been set as 30/365 since we calculate the semivariance at 30-day maturity.

Thus, the raw values of semivariances at the reference (i.e., 30-day) maturity are

$$\sigma_{bull}^2 = w_{near} \times \sigma_{near, bull}^2 + w_{next} \times \sigma_{next, bull}^2$$

$$\sigma_{bear}^2 = w_{near} \times \sigma_{near, bear}^2 + w_{next} \times \sigma_{next, bear}^2$$

which are calculated continuously.

3. Smoothing: Calculate EWMA of raw semivariances

To eliminate the noise in raw semivariances and capture the fundamental trends in the market, an EWMA of raw semivariances is employed as shown below:

$$\begin{aligned}\sigma_{smoothed, bull, t}^2 &= \lambda \times \sigma_{smoothed, bull, t-1}^2 + (1 - \lambda) \times \sigma_{bull, t}^2 \\ \sigma_{smoothed, bear, t}^2 &= \lambda \times \sigma_{smoothed, bear, t-1}^2 + (1 - \lambda) \times \sigma_{bear, t}^2\end{aligned}$$

where $\sigma_{smoothed, s, t-1}^2$ is the previous value of the smoothed semivariance, its weight is the smoothing parameter λ , and $s \in \{bull, bear\}$.

4. Smoothing: Calculate EWMA of raw semivariances

Final step is to convert the smoothed semivariance into *VBULL* and *VBEAR*,

$$\begin{aligned}VBULL &= 100 \times \sqrt{\sigma_{smoothed, bull, t}^2} \\ VBEAR &= 100 \times \sqrt{\sigma_{smoothed, bear, t}^2}\end{aligned}$$

4. Conclusion

This paper introduces a novel approach to measuring risk in cryptocurrency markets by developing Volmex Bull and Bear Implied Volatility Indices (VBULL & VBEAR). These indices distinguish between positive (bull) and negative (bear) return variations, addressing a critical gap in traditional volatility measures that treat all return variations symmetrically.

By separating upside and downside risk, VBULL and VBEAR provide a more complex view of market sentiment, allowing participants to manage asymmetric risks more effectively. This innovation is particularly valuable in highly volatile markets where sharp price movements occur frequently and the risk profiles differ drastically between bullish and bearish conditions.

Overall, the introduction of VBULL and VBEAR represents a significant advancement in the financial modeling of digital assets. These indices provide new insights into market behavior and offer improved frameworks for both risk management and derivative pricing. By incorporating asymmetry in volatility assessment, this methodology opens up new avenues for research and practical applications, potentially reshaping the way risk is quantified and managed in the cryptocurrency space.

References

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- Demeterfi, K., E. Derman, M. Kamal, and J. Zou. "A guide to volatility and variance swaps." *Journal of Derivatives*, 6 (1999), pp. 9-32.